B.Sc. (Hons.) 5th Semester Examination,

December–2022

MATH

Paper–BHM-354

Integral Equations

Time allowed : 3 hours] *[Maximum marks : 60*

Note : Attempt ***five*** *questions in all by selecting at least one question from each section.* ***Question No. 9 is compulsory.***

Section–I

1. (a) Reduce the IVP : $y'' + xy = 1, y(0) = 0, y'(0) = 0$ into Volterra integral equation. 6

(b) Solve the I.E. : $u(x) = f(x) + \lambda \int_a^x k(x, t)\, u(t)\, dt$ by using Method of Successive substitution. 6

2. (a) Solve by using method of Laplace Transform

$$u(x) = 1 - \int_0^x e^{x-t} \cdot u(t)\, dt$$ 6

(b) To find the resolvent kernel to solve the I.E. and hence solve $u(x) = x + \int_0^x (t - x)\, u(t)\, dt$ 6

Section–II

3. (a) To reduce the BVP : $y'' + A(x)y' + B(x)y = g(x)$ $a \le x \le b, y(a) = c_1, y(b) = c_2$ into Fredholm Integral Equation. 6

(b) Explain the method of successive approximation to solve the Fredholm equation of 2nd kind

$$y(x) = f(x) + \lambda \int_a^b k(x, t)\, u(t)\, dt$$ 6

4. (a) Solve the I.E. by method of iterated kernel

$$u(x) = \frac{5}{6}x - \frac{1}{9} + \frac{1}{3}\int_0^1 (x + t)\, u(t)\, dt$$ 6

(b) Solve the I.E. $u(x) = \cos x + \lambda \int_0^{\pi} \sin(x + t)\, u(t)\, dt$ by method of degenerate kernel. 6

Section–III

5. (a) To find Green's Function for B.V.P. : $\frac{d^2u}{dx^2} - u(x) = 0$ with $u(0) = u(1) = 0$. 6

(b) Explain method of variation of parameters to construct the Green's function for Non-Homogeneous linear 2^{nd} degree BVP. 6

6. (a) Reduce the BVP to an integral equation $\frac{d^2u}{dx^2} + \lambda u = x$ with boundary conditions $u(0) = u\left(\frac{\pi}{2}\right) = 0$ using basic properties of Green's function. 6

(b) Explain method of series representation of Green's function in terms of the solutions of the associated homogeneous BVP. 6

Section–IV

7. For the F.I.E. with symmetric kernel :

$y(x) = \lambda \int_a^b k(x,t)\, y(t)\, dt$ show that (i) The eigen function corresponding to two different eigen values are orthogonal over the integral a to b. (ij) the eigen values are real. 12

8. (a) Find the approximation solution to F.I.E.

$$u(x) = \sin x + \int_0^1 (1 - x \cos x t)\, u(t)\, dt$$

by considering its associated Fredholm equation with degenerate kernel. 6

(b) To find the eigen values and eigen functions for

$$y(x) = \lambda \int_0^{\pi} k(x,t)\, y(t)\, dt$$

where $k(x,t) = \begin{cases} \cos x \sin t, & 0 \le x \le t \\ \cos t \sin x, & t \le x \le \pi \end{cases}$ 6

Section–V (Compulsory Question)

9. (a) Explain Linear integral equations with examples. 3

(b) Explain Fredholm Alternative in detail. 3

(c) Write basic properties of Green's function. 3

(d) Define approximation of a kernel by degenerate kernel. 3